An Articulation of Specialized Content Knowledge of Mathematics Teachers in the Context of Measurement Concepts

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Abstract

There has been a consensus in the mathematics education research community that teachers should have a strong mathematics background and an understanding that is deeper than their students' (Ball, Lubienski, Mewborn, 2001). How much mathematics background or understanding is enough for teachers to teach effectively is still a mystery. Part of the early relevant literature points to the importance of the number of mathematics courses teachers have taken whereas the other parts highlight the importance of quality of the knowledge teachers should have. In all these macro level efforts about understanding the nature of teacher knowledge what is missing is an articulation of what exactly it means to teach mathematics effectively and what contributes to that effective teaching of mathematics. The current study aims to contribute to this area.

The current mathematics teacher education literature is covered with studies about categorizing the types of teacher knowledge (e.g., Shulman 1986, 1987; Grossman, 1990; Fennema & Franke, 1992), measuring quality of teacher knowledge (e.g., Ball, 1990; Saiz, 2003), effect of teacher knowledge on student success (e.g., Rowan et al., 2002) or on lesson design (e.g., Hill et al., 2008) and how to improve teacher knowledge (e.g., An & Wu, 2011). The common purpose of these efforts is to know more about the nature of mathematics teacher knowledge and its effect on certain areas such as learning, teaching, and curriculum.

Lee Shulman (1986, 1987) initially described seven categories of teacher knowledge (content knowledge, general pedagogy knowledge, pedagogical content knowledge, curriculum knowledge, knowledge of learners, knowledge of educational contexts, knowledge of educational ends). This work later on evolved into a form called, mathematics knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008) which is the kind of knowledge required for mathematics teachers to have that is specific to teaching. Within the MKT framework, the knowledge required to manage everyday tasks of teaching mathematics (e.g., responding to students' why questions, evaluating students' alternative solutions) is called *Specialized Content Knowledge* (SCK). The current literature falls short in helping us understand the nature of the SCK to be able to respond to the aforesaid everyday tasks of teaching. We know from the literature that such tasks require a particular kind of knowledge that is specific to mathematics teaching. However, what constitutes that knowledge and what contributes to the use of it in actual teaching situations are not clear yet.

Is SCK about conceptual knowledge? Is SCK something beyond conceptual knowledge? We know from the work of Ball and her colleagues that SCK is different from conceptual knowledge, but we do not know what exactly SCK entails especially in teaching certain mathematical ideas. Therefore, in the current study we focused on mathematics teachers' understanding of two tasks of teaching (responding to students' why questions and evaluating students' alternative solutions) that require special knowledge as well as the nature of this knowledge in the context of measurement concepts through gathering data from in-service mathematics teachers with a variety of experience. The purpose of this study is to provide an articulation of the meaning of SCK in the context of

measurement concepts (i.e., length, area, volume). More specifically we aim to model mathematics teachers' ways of operating with an SCK and to identify contributing factors to their operating. We particularly focused on SCK since there is not fulfilling micro-level analysis of SCK in the relevant literature. We also chose to focus on measurement concepts since there is a scarcity of research in the area of measurement in spite of its richness (Outhred & Mitchelmore, 2000). We only provide an example for volume concept in this paper because of word limitations.

Method

It is a qualitative study conducted with 12 in-service mathematics teachers (grades 5-8) teaching 12-14-year-olds. The volunteers (4 females, 8 males, with 10-20 years of teaching experience) were chosen through convenience sampling.

The data, derived from Bayram (2016), was collected through two-hour one-on-one semi-structured interviews that were videotaped. During the interviews the participants were asked four open-ended questions about scenarios of student work regarding length, area and volume concepts. The interviews focused on teachers' knowledge of *responding to students' why questions* and *evaluating students' alternative solutions*. In order to prevent participants from catching a pattern among the questions, one of us met each teacher several times and applied each question one at a time.

A sample question is provided below, the purpose of which was to understand the kind of resources teachers drew on as they operated on *students' alternative solutions*. The mathematics of the scenario was adapted from Battista (2003). It was given the teachers with a supporting diagram with fewer explanations than below:

> *A teacher made a number of mini packages made up of 2 interlocking unit cubes. She then asked her students to figure out how many such packages would fit into a given open-top rectangular right prism. Each student was given a single package (consisting of two-unit cubes interlocked without any trace) and a transparent prism. Here is a common solution used by the majority of the class: The students first located the given single package at a bottom corner of the given prism, and then slide it along the length of the prism by making necessary markings at each end of the package as they slide the package. They then counted that there were 6 packages that fit along the length of the prism. Students then moved the package back to the same bottom corner of the prism again, rotated it 90 degrees to the left, and slide it along the width of the prism, which results in 3 packages. The students finally moved the package back to the original bottom corner again, rotated it 90 degrees up in the z-axis direction, and slide it along the height, which results in 4 packages. Once they found 6 packages for length, 3 packages for width and 4 packages for height, they did (6×3)×4=72, and said, "volume is 72." Explain on what basis such solution method is valid or not.*

The data was analysed through qualitative techniques.

Conclusions

Our analysis of the data suggests that participant teachers operate in two distinct ways in approaching the given volume task:

(1) The teachers compared their current understanding of volume concept with the students' proposed solutions, which results in insufficient analysis of students' alternative solutions (SCK) on

their part. For instance, in finding the volume of the given prism, none of the participants initially focused on even the possibility that changing the orientation of the given package (package of two 1cm×1cm×1cm units) would affect the total number of packages fitting in the given prism. They all approved the given student solution of multiplying three dimensional measures $(6\times3)\times4=72$ by operating from the formula, *length* × *width* × *height*.

(2) The teachers also drew on the appropriate/required mathematical knowledge by referring to the question of, "On what mathematical basis the proposed solution is valid?" Such efforts result in sufficient analysis on teachers' part. For example, after their initial attempt of comparing their own understanding of volume with the given solution, later on during the interview, all the participant teachers changed such initial approval about the given solution method of students. This is because they realized that the total number of packages fitting in the given prism is not 72 during their deep thinking (with researcher's probing) about the given student solution, or during their calculation of the volume of the prism by considering each package as 2-unit items and counting them, or as a result of the researcher's leading/probing questions. Though such realization in one of these three categories only came after lots of drill, leading, or probing.

As a result we can conclude that everyday task of "analysing students' alternative solution strategies" (SCK) necessitates a special kind of **questioning** linked to the required **mathematical knowledge** that requires more than pure mathematical knowledge.

References

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education,7(2),* 145-172.

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education, 21*(2), 132-144.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V.Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). New York: Macmillan.

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Battista, M. T. (2003). Understanding students' thinking about area and volume measurement. In D. H. Clements & G. Bright (Eds.), *Learning and Teaching measurement: 2003 Yearbook* (pp.122-142). Reston, VA: National Council of Teachers of Mathematics.

Bayram, E. (2016). *The nature of middle school mathematics teachers' specialized content knowledge in the context of measurement* (master's thesis). Mevlana University, Konya, Turkey.

Fennema, E., & Franke , M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics (pp. 147-164). New York: Macmillan.

Grossman, P. L. (1990). The making of a teacher: Teacher knowledge and teacher education. New York: Teachers College Press

Hill, H. C., Blunk, M., Charalambous, C., Lewis, J., Phelps, G., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26(4)*, 430-511.

Outhred, L., & Mitchelmore, M. C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education, 31*, 144-167.

Rowan, B. & Correnti, R. & Miller, R. (2002). What large-scale, survey research tells us about teacher effects on student achievement: Insights from the Prospects study of elementary schools. *Teachers College Record*, *104*(8), 1525-1567.

Sáiz, M. (2003). Primary teachers' conceptions about the concept of volume: The case of volume-measurable objects. *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th Conference of PME-NA* (vol.4, pp.95-102). Honolulu, HI: PME-NA

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *American Educational Research Association, 15*(2), 4-14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review,* 57(1), 1-23.